

# Math 6010 - Assignment 6

Due March 1, 2021

- (1) Show that the Diagonal Halting Problem

$$K := \{x \in \mathbb{N} \mid \varphi_x(x) \text{ is defined} \}$$

is computably enumerable but not computable.

- (2,3) Show that  $\Sigma_n^0$  for  $n \in \mathbb{N}$  is closed under  $\wedge, \vee$  and bounded quantifiers.

- (4,5) Prove that the following are equivalent for  $A \subseteq \mathbb{N}$ :

- (a)  $A$  is computably enumerable.
- (b)  $A$  is the domain of some (partial) computable function.
- (c)  $A$  is in  $\Sigma_1^0$ .
- (d)  $A$  is the range of a (partial) computable function  $f: \mathbb{N} \rightarrow \mathbb{N}$ .