Math 6010 - Assignment 2

Due February 3, 2021

(1) Design a Turing machine that computes $n+1$ for an input n in binary.

   More precisely the machine starts with the input n in reverse binary, e.g., $n = 2$ is represented by the starting configuration $(s, 01\llcorner, 0)$. The machine should halt with only $n + 1$ in reverse binary on its tape and the tape head pointing at its leading digit, e.g., in the final configuration $(t, 01\llcorner, 1)$ for $n = 2$.

   List the configurations of your machine when given the input $n = 5$.

(2) Reading assignment: Read pages 148-150 in [1] (available in the Math library) on multi-tape Turing machines and why they are equivalent to single-tape machines.

(3) A Turing machine with bi-infinite tape is similar to the machine we defined in class except that the tape extends infinitely to the left as well. At the start of computation the input is written somewhere on the tape, left and right of it there are only blanks, and the tape head is at the leftmost position of the input.

   Show that Turing machines with bi-infinite tape accept the same languages as the machines defined in class.

(4) * Design a Turing machine that recognizes $L = \{1^{n^2} \mid n \in \mathbb{N}\}$. [Choose an appropriate tape alphabet. Feel free to use any variation of Turing machines that you want.]

REFERENCES

[1] Sipser, Michael. Introduction to the theory of computation. Thomson Course Technology, Boston, 2nd edition, 2006.