Math 6010 - Assignment 10

Due November 13, 2023

(1) Show that as class of languages P is closed under the following operations:
   (a) union,
   (b) concatenation,
   (c) Kleene star,
   (d) complement.

(2) (a) Show that NP is closed under union, concatenation and Kleene star.
   (b) It is not known whether NP is closed under complementation. Explain why the naive proof for this does not work.

(3) Graphs $G = (V, E)$ and $H = (W, F)$ are isomorphic if there exists a bijection $f\colon V \to W$ such that for all $i, j \in V$: $(i, j) \in E$ iff $(f(i), f(j)) \in F$.

Show that

$$\text{GraphIsomorphism} := \{(G, H) \mid G, H \text{ are isomorphic graphs}\}$$

is in NP.

If you've seen groups in some algebra course, do (4); else do (5).

(4) Show that

$$\text{Primes} := \{n \mid n \text{ is a prime in binary}\}$$

is in NP.

Use the following fact: For $n > 1$, the multiplicative group $\mathbb{Z}_n^* := \{x \in \mathbb{Z}_n \mid x \text{ has a multiplicative inverse}\}$ is cyclic of order $n - 1$ iff n is prime. Then obtain a witness for primality of n from the prime factors of $n - 1$.

(5) A graph $G = (V, E)$ is undirected if E is a symmetric relation. For $k \in \mathbb{N}$, a k-**coloring** of an undirected graph G is a function $f\colon V \to \{1, \ldots, k\}$ such that

$$\forall i, j \in V :\ (i, j) \in E \Rightarrow f(i) \neq f(j).$$

Hence vertices connected by an edge are assigned different colors.

Show that

$$\text{2-coloring} := \{G \mid G \text{ is an undirected graph with a 2-coloring}\}$$

is in P.