# Math 6010 - Assignment 8

Due October 30, 2023

(1) Show that there are (at least) continuum many distinct Turing degrees.
(2) Prove the **Relativized Complementation Theorem**:
$B \leq_T A$ iff $B$ and $\bar{B}$ are computably enumerable in $A$.
(3) Show that $\deg(A \oplus B)$ is the least upper bound for $\deg(A)$ and $\deg(B)$.
(4) Let $A \subseteq \mathbb{N}$. Show
$$A <_T A'$$
(i.e. $A$ is Turing reducible to its jump $A' = \{x \mid \varphi_x^A(x) \downarrow\}$ but not conversely).
(5) Let $A, B \subseteq \mathbb{N}$. Prove
$$B \leq_T A \text{ iff } B' \leq_m A'.$$
Hint: For the "if"-direction show that $B$ and $\bar{B}$ are $A$-computably enumerable using the Relativized Parameter and Enumeration Theorems.