

Math 6010 - Assignment 6

Due October 16, 2023

- (1) Prove that the following are equivalent for $A \subseteq \mathbb{N}$ (for those implications that were not covered before):
- (a) A is computably enumerable.
 - (b) A is the domain of some (partial) computable function.
 - (c) A is in Σ_1^0 .
 - (d) A is the range of a (partial) computable function $f: \mathbb{N} \rightarrow \mathbb{N}$.

- (2) For $n \in \mathbb{N}$ let $\Delta_n^0 := \Sigma_n^0 \cap \Pi_n^0$. Show that

$$\Sigma_n^0 \cup \Pi_n^0 \subset \Delta_{n+1}^0$$

for $n \geq 1$. What about $n = 0$?

Hint: To show that the containment is proper let P be Σ_n^0 but not Π_n^0 and consider

$$Q(x, z) := (P(x) \wedge z = 0) \vee (\neg P(x) \wedge z = 1).$$

- (3,4) Let $W_e := \text{dom } \varphi_e^{(1)}$.

$$K := \{x \mid x \in W_x\}$$

$$H := \{x \mid 0 \in W_x\}$$

$$T := \{x \mid W_x = \mathbb{N}\}$$

$$F := \{x \mid W_x \text{ is finite}\}$$

Show that K, H, T, F are all complete for appropriate levels in the arithmetical hierarchy (for those that were not covered in class yet).

- (5) Which of the sets K, H, T, F and their complements are many-one reducible to each other?