Math 6010 - Assignment 4

Due October 2, 2023

(1) For $A \leq_m B$ show:
If B is computable, c.e., co-c.e., respectively, then so is A.

(2) Show that a language A is c.e. iff $A \leq_m \mathrm{AP}$.

(3) Is the set of Turing machines that accept the empty language

$$\{[M] \mid L(M) = \emptyset\}$$

computably enumerable? What about its complement?

(4) Show that the set of codes for Turing machines that write some nonblank symbol eventually when started with empty input is computable.
Why does this not contradict Rice's Theorem?
What about machines that write the letter 0 sometime after starting on an empty tape?

(5) Show that the following functions are primitive recursive:
   (a) monus $x \dot{-} y := \begin{cases} x - y & \text{if } x > y \\ 0 & \text{else} \end{cases}$
   (b) $f(\bar{x}, y) := \sum_{z \leq y} g(\bar{x}, z)$ for every primitive recursive function g