

Math 6010 - Assignment 3

Due September 25, 2023

- (1) Prove or disprove:
 - (a) For computably enumerable languages A, B , also AB is c.e.
 - (b) For A, B c.e., also $A \setminus B$ is c.e.
- (2) Complete the proof from class:
 - (a) A partial function $f: \Sigma^* \rightarrow_p \Sigma^*$ is computable iff its graph
$$L_f = \{(x, y) \in \Sigma^* \times \Sigma^* \mid x \in \text{domain}(f), f(x) = y\}$$
 is computably enumerable.
 - (b) If a total function is computable, then its graph is computable.
- (3) Argue that the language of multivariate polynomials (in some appropriate coding) over the integers that have an integer root is computably enumerable.
- (4) Show that
$$\text{succ} := \lambda n f x. f((nf)x)$$
 defines the successor function on natural numbers (Church numerals) in λ -calculus.
- (5) Give many-one reductions from the halting problem HP to the acceptance problem AP and conversely.