Math 6010 - Assignment 1

Due September 11, 2023

(1) Construct a deterministic finite automaton over $\Sigma := \{0, 1\}$ that accepts all positive multiples of 3 in binary (that is, all strings starting in 1 that represent a positive integer divisible by 3).

(2) (a) [1, Exercise 3.1.1 a] Give a regular expression for the language over $\{a, b, c\}$ containing at least one a and one b.

(b) Give a regular expression for the language of words over $\{0, 1\}$ in which the number of 0s is a multiple of 3.

(3) [1, Exercise 4.1.2 a] Show that $\{1^{n^2} \mid n \in \mathbb{N}\}$ is not a regular language.

(4) Show that the class of regular languages on an alphabet Σ is closed under complementation, union, intersection, concatenation, and Kleene star $*$.

(Hint: Use the characterization of regular languages by automata whenever convenient.)

(5) Let L be the language over $0, 1$ that contains every word with 0101 as substring. Show that any DFA with language L has at least 5 states.

References

[1] Hopcroft, John; Motwani, Rajeev; Ullman, Jeffrey. Introduction to automata theory, languages, and computation. Pearson, 3rd edition, 2006.