32. The number of countable models

Let T be a complete theory in countable language with infinite models. Let κ be an infinite cardinal.

What are the possible values for
$I(T, \kappa) :=$ number of non-isomorphic models of T of cardinality κ

Example

- $I(T, \kappa) = 1$ for κ-categorical T (e.g. DLO)
- $I(\mathsf{ACF}_p, \aleph_0) = \aleph_0$
- $I(T, \kappa) = 2^{\aleph_0}$ if $S_n(T) = 2^{\aleph_0}$ (e.g. RCF, $\mathrm{Th}(\mathbb{N})$)

A theory with 3 countable models

Over the language $\{<, c_0, c_1, \dots\}$, let

$$T_3 := \text{ DLO } \cup \{c_n < c_m \mid n < m < \omega\}.$$

The countable models of T_3 are $\mathcal{A} \cong (\mathbb{Q}, <, c_0, c_1, \dots)$:

atomic: (c_n) has no upper bound in $\mathcal{A}$, i.e., $\lim_{n\to\infty} c_n =$

saturated: (c_n) has an upper bound but no least upper bound, i.e. $\lim_{n\to\infty} c_n =$

neither: (c_n) has a least upper bound, i.e. $\lim_{n\to\infty} c_n =$

Marker, Ex. 2.5.28: T_3 is complete and has expansions T_n with exactly $n \geq 3$ countable models.

Question

What about the gaps?

- $I(T, \aleph_0) = 2$,
- $\aleph_0 < I(T, \aleph_0) < 2^{\aleph_0}$

See Mattan's talk.