Math 6000: Model Theory

CU Boulder, Spring 2026

What is it?

Model theory studies mathematical structures by the statements they satisfy as well as subsets definable within structures.

Chang, Keisler: **model theory = universal algebra + logic**
Hodges: **model theory = algebraic geometry - fields**

What is it good for?

Typical questions:

What is true/definable in a concrete structure?

Is the given statement true over the integers? The reals?

$$\forall x \; \exists y \; (x = 0 \vee xy = 1)$$

What can be said about the structures satisfying specific statements?

Is every model for the theory of the field of reals isomorphic to $\mathbb{R}$? What about $\mathbb{C}$?

Some necessary formalizations

- **Structures** are sets with operations and relations defined on them, e.g.,
  $(\mathbb{R}, +, -, 0, \cdot, 1, <)$
- Statements are given in **first-order logic**, e.g.,
  $\forall x \ (0 < x \to \exists y \ x = y^2)$

syntax	**semantics**
languages	structures
terms	evaluations
formulas	satisfaction
theories	models
provability ($\vdash$)	entailment ($\models$)
proof theory	**model theory**

Course content

- Compactness Theorem
- Löwenheim-Skolem Theorems
- Categoricity
- Ehrenfeucht-Fraïssé Games
- Quantifier Eliminiation
- Types
- Saturated Models
- Morley's Categoricity Theorem

Text. David Marker, Model Theory: An Introduction (2002).