Math 2001 - Assignment 11

Due April 8, 2026

(1) Let T be a completion of the theory of fields with an infinite model. Show that T is not ω-categorical.

Hint: Use the Ryll-Nardzewski Theorem and show that the formulas $y = x^n$ for $n \in \mathbb{N}$ are inequivalent modulo T.

(2) Characterize ω-categorical vector spaces.

Hint: Consider a countable $\mathcal{F}$-vector space V and distinguish whether the field $\mathcal{F}$ is finite/countable.

(3) Show that there exists a countable bipartite graph that embeds all countable bipartite graphs.

Hint: The vertex set of a bipartite graph splits into two disjoint subsets $U_!$ and U_2 such that there are no edges between any two vertices in U_i for any $i \in \{1, 2\}$.

Bipartite graphs can be considered as relational structures over the language $\mathcal{L} = \{E, U\}$ where E is the binary edge relation and U is an equivalence relation with at most two classes (namely, U_1, U_2) such that

$$\forall x, y\ U(x, y) \to \neg E(x, y).$$

Argue that the class K of finite bipartite graphs is Fraïssé (no need to prove the properties in detail).

Show that the Fraïssé limit of K (the generic bipartite graph) embeds all countable bipartite graphs.

(4) Show that if $\mathcal{A}$ is ω-categorical, then the expansion $(\mathcal{A}, b)$ of $\mathcal{A}$ by a constant b is also ω-categorical.