Math 2001 - Assignment 10

Due April 1, 2026

(1) Show that every finite model is ω-saturated.

(2) Let $\mathcal{L}$ be the language consisting only of constant symbols c_n for $n < \omega$, let T be the theory with axioms $c_m \neq c_n$ for $m < n < \omega$.

  (a) Characterize the countable models of T up to isomorphism [Hint: There are only countably many].

  (b) Does T have a countable atomic model? What is it?

  (c) Does T have a saturated countable model? What is it?

(3) [1, Exercise 4.5.17] Show

  (a) Every algebraically closed field is homogenous.

  (b) Every uncountable algebraically closed field is saturated. What about countable algebraically closed fields?

(4) Let $\mathcal{A}$ be a structure, $B \subseteq A$ and $f\colon B \to A$ a partial elementary map. For $a \in A$ show that

$$\{\varphi(x, f(\bar{b})) \ : \ \mathcal{A} \models \varphi(a, \bar{b}), \bar{b} \text{ a tuple over } B\}$$

is a type, denoted $f(\mathrm{tp}^{\mathcal{A}}(a/B))$.

References

[1] Marker. Model Theory: An Introduction. Springer, 2002.