Math 2001 - Assignment 8

Due March 11, 2026

(1) Show that a structure $\mathcal{A}$ realizes all 1-types over A if and only if $\mathcal{A}$ realizes all n-types over A.
(2) [1, Ex 4.5.7] Let T be the theory of $(\mathbb{N}, +, \cdot, 0, 1)$. Show that $|S_1(T)| = 2^{\aleph_0}$.

    Hint: Let p_n be the n-the prime number. For $A \subseteq \mathbb{N}$, consider $\Gamma_A(x) := \{p_n \text{ divides } x | n \in A\} \cup \{p_n \text{ does not divide } x | n \notin A\}$.
(3) Show for any structure $\mathcal{A}$ that the expansion $\mathcal{A}_A$ is an atomic model of $\mathrm{Th}(\mathcal{A}_A)$.
(4) [2, Ex 4.5.3] Let T be the theory of $(\mathbb{R}, <, Q)$ where Q is a unary predicate for the rational numbers. Does T have a prime model?

References

[1] Marker. Model Theory: An Introduction. Springer, 2002.
[2] Tent, Ziegler. A Course in Model Theory. Lecture Notes in Logic, Cambridge, 2012.