Math 2001 - Assignment 2

Due January 28, 2026

(1) Complete the steps missing from the proof that if $\mathcal{A} \cong \mathcal{B}$, then $\mathcal{A} \equiv \mathcal{B}$ from the slides for January 21.
(2) Let T be a satisfiable theory. Show that T is complete if all models of T are elementary equivalent.
(3) [1, Exercise 1.4.3] Let $\mathcal{L}$ be any countable language. Show that for any infinite cardinal κ there are at most 2^κ non-isomorphic $\mathcal{L}$-structures of cardinality κ.
(Hint: Some basic cardinal arithmetic needed.)
(4) [1, Exercise 1.4.8] Let $\mathcal{L} = \{+, 0\}$. Show that $\mathbb{Z} \times \mathbb{Z} \not\equiv \mathbb{Z}$.
(5) Let T be a theory that has arbitrarily large finite models. Show that T has an infinite model.

References

[1] David Marker. Model Theory: An Introduction. Springer, 2002.