Math 4140 - Assignment 11

Due April 15, 2024

(1) For ψ, χ irreducible characters of a finite group G such that $\psi(1) = 1$, show that $\psi\chi$ is irreducible.
(2) [1, Exercise 19.2]
(3) [1, Exercise 19.5]
(4) Give the character table of $S_3 \times S_3$.

Let

$$D_{12} = \langle a, b \ : \ a^6 = 1, b^2 = 1, b^{-1}ab = a^{-1} \rangle,$$
$$C_6 = \langle a \ : \ a^6 = 1 \rangle$$

be presentations of the dihedral group of order 12, its cyclic subgroup of order 6, respectively, with character tables ($\omega = e^{2\pi i/6}$):

D_{12}	1	a	a^2	a^3	b	ab
χ_1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1
χ_3	1	-1	1	-1	1	-1
χ_4	1	-1	1	-1	-1	1
χ_5	2	1	-1	-2	0	0
χ_6	2	-1	-1	2	0	0

C_6	1	a	a^2	a^3	a^4	a^5
ψ_1	1	1	1	1	1	1
ψ_2	1	ω	ω^2	-1	ω^4	ω^5
ψ_3	1	ω^2	ω^4	1	ω^2	ω^4
ψ_4	1	-1	1	-1	1	-1
ψ_5	1	ω^4	ω^2	1	ω^4	ω^2
ψ_6	1	ω^5	ω^4	-1	ω^2	ω

(5) Express $\chi_i|_{C_6}$ as linear combinations as the ψ_j for all $i \leq 6$.
(6) (a) Show that $H := \ker\chi_3$ is a subgroup of order 6.
    (b) Show that $\chi_i|_H$ is irreducible for all $i \leq 6$.
    (c) Give the character table of H.
    (d) Which group is H isomorphic to?
(7) Show that the nontrivial proper normal subgroups of D_{12} are exactly
    - three subgroups of order 6,
    - the derived subgroup of order 3,
    - the center of order 2.

    Draw the inclusions of these subgroups. Which of the subgroups of order 6 are abelian?

References

[1] G. James and M. Liebeck. Representations and characters of groups. Cambridge University Press, second edition, 2001.