Math 3140 Spring 2024 - Review

1. **Groups.**
   (1) axioms, uniqueness of identity and inverses (2), subgroups, order of elements, generators (3), cyclic groups and their subgroups (4), isomorphisms, (inner) automorphism group, Cayley's Theorem (6)
   (2) $\mathbb{Z}_n$, dihedral groups D_{2n} with generators and relations (1), symmetric groups S_n, cycle notation of permutations and their order (5), sign of permutations, alternating group A_n (5), general and special linear group $\mathrm{GL}(n, F)$, groups of order p^2 and pq up to isomorphism (24.)
   (3) left cosets, Lagrange's Theorem, index of subgroups (7)
   (4) conjugacy, normal subgroups, factor groups (9), center, derived (commutator) subgroup, simple groups (25)
   (5) kernels, images of homomorphisms, first isomorphism theorem, correspondence theorem (10)
   (6) external (8) and internal direct products (9), elements and subgroups of specific order in direct products, Fundamental Theorem of finite abelian groups (11)
   (7) group actions, orbits, transitivity, stabilizer, fixed points (29), Orbit-Stabilizer Theorem (5), counting permutations of fixed cycle structure (5), counting orbits, Burnside-Frobenius Lemma (29)
   (8) conjugacy classes, class equation, p-groups have non-trivial center, Sylow subgroups, Sylow Theorems 1-3 (24)

2. **Rings.**
   (1) axioms, commutative ring with 1, units, subrings (12), integral domains, fields (13), direct product
   (2) ideals, quotient rings, maximal ideals (14)
   (3) kernels, images of homomorphisms, first isomorphism theorem (15)
   (4) matrix ring $R^{n\times n}$, polynomial ring $R[x]$, fields $\mathbb{Z}_p$ (p prime), $\mathbb{Q}, \mathbb{R}, \mathbb{C}$