Math 3140 - Assignment 11

Due April 17, 2024

(1) Show every finite abelian group is the direct product of its Sylow subgroups.

(2) Show that all Sylow p-subgroups of a finite group G for a prime p are isomorphic.

(3) (a) Find a Sylow 2-subgroup of S_4. Show that it is isomorphic to D_8. How many Sylow 2-subgroups are there?
   (b) Find all Sylow 3-subgroup of S_4.
   (c) Find all Sylow 5-subgroups of S_4

(4) Let $n \in \mathbb{N}$ be odd.
   (a) Give a Sylow 2-subgroup of D_{2n}. What is it isomorphic to? How many are there?
   (b) Let p be an odd prime. What are the Sylow p-subgroups of D_{2n}?
   (c) Are any of the Sylow subgroups of D_{2n} normal?

(5) For every prime p give a Sylow p-subgroup of A_5. Can you determine how many there are for each p? Are any of them normal?

   Hint: Recall the number of permutations of a certain cycle structure.

(6) (a) Show that every group of order 56 has a proper non-trivial normal subgroup.
   (b) Show that every group of order 175 is abelian.

   Hint: Determine the numbers of Sylow subgroups.

(7) (a) How many groups of size 21 are there up to isomorphism? What do they look like?
   (b) How many groups of size 33?