Math 3130 - Assignment 1

Due January 24, 2024

(1) Complete the multiplication table for the symmetries $1, a, b, c$ of a (nonsquare) rectangle.

A multiplication is *commutative* if order of the arguments does not matter, that is, $xy = yx$ for all x and y.

Is the multiplication of symmetries of a rectangle commutative?

(2) With pictures and words describe the set of symmetries of an equilateral triangle.

How many symmetries are there?

For each symmetry, give its inverse.

Is the multiplication of symmetries of an equilateral triangle commutative?

(3) A *regular n-gon* is a plane figure that has n sides of equal length and n equal angles. E.g., for $n = 4$ a regular n-gon is just a square.

(a) Describe the symmetries of the regular n-gon ($n \geq 3$).
Hint: Consider the cases of n even and n odd separately.

(b) How many symmetries are there for a regular n-gon ($n \geq 3$)?

The group of symmetries of a regular n-gon is called the *dihedral group of order* $2n$ and denoted D_{2n} for short.

Attention: In [1] and some other books this is denoted D_n instead.

(4) (a) In D_{2n}, describe geometrically why the composition of two rotations is a rotation.

(b) In D_{2n}, describe geometrically why the composition of two reflections is a rotation.
Hint: What happens with the positions of two neighboring corners after one reflection? After two?

(5) Let $n \in \mathbb{N}, a, a', b, b' \in \mathbb{Z}$ such that $a \equiv_n a', b \equiv_n b'$. Show

$$a + b \equiv_n a' + b'.$$

(6) (a) Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group.

(b) Show that $(\mathbb{Z}_5 \setminus \{0\}, \cdot)$ is a group.

(7) Let $n \geq 2$ and a be integers. Show that $ax \mod n = 1$ has a solution iff $\gcd(a, n) = 1$.

References

[1] Joseph A. Gallian. Contemporary Abstract Algebra. Houghton Mifflin Company, sixth edition, 2006.