Math 3140 Fall 2026 - Review for Midterm 2

Numbers refer to Sections in Gallian, Contemporary Abstract Algebra.

1. **Consequences of Lagrange's Theorem.**
   - Every group of prime order is cyclic (Cor 3 of Thm 7.1)
   - For p prime, every group of order $2p$ is isomorphic to $\mathbb{Z}_{2p}$ or D_{2p} (Thm 7.2)
   - Orbit-Stabilizer Theorem (Thm 7.3)
2. **Structure of groups.**
   - External (8) and internal direct products (9)
   - Elements of specific order in direct products (Thm 8.1)
   - $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic iff $\gcd(m, n) = 1$ (Thm 8.2)
   - Fundamental Theorem of finite abelian groups (Thm 11.1)
   - Isomorphism classes of finite abelian groups (11)
3. **Normal subgroups and factor groups.**
   - Homomorphisms, kernels and images (10).
   - Normal subgroups (9) are the same as kernels of homomorphisms (Thms 10.2, 10.4).
   - Examples of normal subgroups: center $Z(G)$, commutator subgroup G'
   - Sign of permutations; alternating group A_n is normal in S_n (Thms 5.4-5.7)
   - Conjugacy classes
   - Permutations are conjugate in S_n iff they have the same cycle structure.
   - Factor groups (Thm 9.2)
   - First Isomorphism Theorem (Thm 10.3)
   - Correspondence Theorem (cf. Thm 10.2)