

Math 3140 - Assignment 7

Due October 9, 2026

- (1) The **exponent** of a group G is the smallest $n > 0$ such that $g^n = 1$ for all $g \in G$ if it exists; else the exponent of G is infinite.

Show that every group G of exponent 2 is abelian.

- (2) Let $(G, \cdot)$ be a group and $S \subseteq G$. Show that the subgroup generated by S consists of all finite products of integer powers of elements in S , i.e.

$$\langle S \rangle = \{a_1^{k_1} \cdots a_n^{k_n} : n \geq 0, k_1, \dots, k_n \in \mathbb{Z}, a_1, \dots, a_n \in S\}$$

Hint: Show that if a subgroup H contains S , then it also contains all products of powers of elements in S .

Conversely, show that the set of products of powers of elements in S is a subgroup of G .

- (3) For G a permutation group on X and $x \in X$, the **stabilizer** of x in G is

$$\text{stab}_G(x) := \{g \in G : g(x) = x\},$$

the set of all permutations that fix x .

Show that $\text{stab}_G(x)$ is a subgroup of G .

- (4) For the group $G = \{(), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}$:

- (a) Find the stabilizer of 1 and the orbit of 1.
- (b) Find the stabilizer of 3 and the orbit of 3.
- (c) Find the stabilizer of 5 and the orbit of 5.

- (5) Give an isomorphism between the following pairs of groups or explain why they are not isomorphic.

- (a) $\mathbb{Z}_8 \times \mathbb{Z}_2, \mathbb{Z}_4 \times \mathbb{Z}_4$
- (b) $\mathbb{Z}_{12}, \mathbb{Z}_4 \times \mathbb{Z}_3$
- (c) $\mathbb{Z}_4 \times \mathbb{Z}_2, D_8$

- (6) Let p be a prime. Show that $\mathbb{Z}_p \times \mathbb{Z}_p$ has exactly $p+1$ subgroups of order p .

Give one generator for each subgroup of order p .

- (7) For a group G and $x, y \in G$ say x is **conjugate** to y in G if

$$\exists g \in G : y = gxg^{-1}.$$

Show that conjugacy is an equivalence relation.

Its equivalence classes are called the **conjugacy classes** of G .