

Math 3140 - Assignment 6

Due October 2, 2026

- (1) For the following subgroups H of G , find all the left cosets of H in G . Give one representative for each left coset. How many are there?
- (a) $G = \mathbb{R}^2$ under addition, $H = \{(x, 0) : x \in \mathbb{R}\}$
 - (b) $G = \langle a \rangle$ of order 12, $H = \langle a^4 \rangle$
 - (c) $G = \mathbb{R}^*$ under multiplication, $H = \mathbb{R}^+$ the subgroup of positive reals
- (2) Let G be a finite group with subgroups $H \leq K \leq G$. Show that

$$|G : H| = |G : K| \cdot |K : H|.$$

- (3) For groups $(G, \cdot_G)$ and $(H, \cdot_H)$ show that the *direct product*

$$G \times H := \{(g, h) : g \in G, h \in H\}$$

under the operation

$$(g_1, h_1) \cdot (g_2, h_2) := (g_1 \cdot_G g_2, h_1 \cdot_H h_2)$$

for $g_1, g_2 \in G, h_1, h_2 \in H$ is a group.

- (4) Give an isomorphism between the following pairs of groups or explain why they are not isomorphic.
- (a) $G \times H, H \times G$
 - (b) $\mathbb{C}, \mathbb{R} \times \mathbb{R}$ under addition
 - (c) $\mathbb{C}^*, \mathbb{R}^* \times \mathbb{R}^*$ under multiplication
 - (d) $\mathbb{Z} \times \mathbb{Z}, (\{2^x 3^y : x, y \in \mathbb{Z}\}, \cdot)$
- (5) Note that $1 \times H$ is a subgroup of $G \times H$. Find all the left cosets of $1 \times H$ in $G \times H$. Give one representative for each left coset. How many are there?
- (6) Let G be a nontrivial group that has no proper, nontrivial subgroups (i.e. 1 and G are the only subgroups of G). Show that $|G|$ is prime.

Hint: Do not assume at the outset that G is finite.