

Math 3140 - Assignment 5

Due September 25, 2026

- (1) Let $\mathbb{C}$ be the set of complex numbers and

$$M = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}.$$

Show that $(\mathbb{C}, +) \cong (M, +)$ and $(\mathbb{C} \setminus \{0\}, \cdot) \cong (M \setminus \{0\}, \cdot)$.

- (2) Let G be a group. Show that $\text{Aut}G$ is a group under composition of functions.
- (3) For a group G and $g \in G$, define the inner automorphism

$$\varphi_g: G \rightarrow G, \quad x \mapsto gxg^{-1}.$$

Show

- (a) $\varphi_g \in \text{Aut}G$.
- (b) $\Phi: G \rightarrow \text{Aut}G, \quad g \mapsto \varphi_g$, is a homomorphism.
- (c) $\ker \Phi = Z(G)$.
- (4) Let $D_8 = \{1, a, a^2, a^3, b, ab, a^2b, a^3b\}$ be the dihedral group of order 8 generated by a rotation a and a reflection b .
- (a) When are $\varphi_g = \varphi_h$ for $g, h \in D_8$?
- (b) What is the order of $\text{Inn}D_8$?
- (c) List all distinct elements in $\text{Inn}D_8$.
- (5) Show that $|\text{Aut}D_8| \leq 8$.

Hint: Explain why an automorphism is uniquely determined by what it does to the generators a and b . Use the properties of automorphisms to restrict where could these be mapped to.

- (6) Find non-isomorphic groups G, H such that $\text{Aut}G \cong \text{Aut}H$.
- (7) Let $(G, \cdot)$ be a group and $g \in G$. Show that the translation

$$a_g: G \rightarrow G, \quad x \mapsto gx,$$

is a permutation on G . Is a_g a homomorphism on G ?