Math 3140 - Assignment 4

Due September 18, 2026

(1) For permutations $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$ compute

$$\alpha^{-1}, \alpha\beta, \beta\alpha$$

and their orders.

(2) Find all subgroups of S_3, give generators for each and show their inclusions in a subgroup lattice.

Hint: First find all cyclic subgroups. Then all subgroups that need two generators, etc.

(3) Show that a cycle of length ℓ can be written as a product of $\ell - 1$ transpositions,

$$(a_1\ a_2\ \ldots a_\ell) = (a_1\ a_\ell)(a_1 a_{\ell-1}) \ldots (a_1 a_2).$$

(4) Show that $Z(S_n) = 1$ for $n \geq 3$.

(5) Let $(G, \cdot, ^{-1}, 1_G)$ and $(H, *, ^{-1}, 1_H)$ be groups with identities $1_G, 1_H$, respectively. Let $\varphi\colon G \to H$ satisfy

$$\varphi(x \cdot y) = \varphi(x) * \varphi(y) \text{ for all } x, y \in G.$$

Show:

(a) $\varphi(1_G) = 1_H$.

Hint: Start by evaluating $\varphi(1_G \cdot 1_G)$ in two ways.

(b) $\varphi(x^{-1}) = \varphi(x)^{-1}$ for all $x \in G$.

Hint: Use (a).

(6) Let G, H be groups and $\varphi\colon G \to H$ a homomorphism. Show

(a) The image of φ,

$$\varphi(G) := \{\varphi(g) \ : \ g \in G\}$$

is a subgroup of H.

(b) The kernel of φ,

$$\ker \varphi := \{g \in G \ : \ \varphi(g) = 1\}$$

is a subgroup of G.

Recall image and kernel of linear maps on vector spaces.