Math 2135 Fall 2026 - Review for Midterm 2

Numbers refer to sections in Lay et al., Linear algebra and its applications.

1. **Vector spaces.**
   (1) subspaces (2.8, 4.1)
   (2) spans of vectors and null spaces of matrices are subspaces (2.8, 4.2)

2. **Basis of a vector space.**
   (1) dimension (4.5), Basis Theorem (cf. 2.3.1)
   (2) Spanning Set Theorem to reduce a spanning set to a basis (2.8, 4.3), extend a linear independent set to a basis (4.5)
   (3) basis and dimension for column space, row space, null space of a matrix (2.8, 4.3)
   (4) coordinates with respect to a basis B (2.9, 4.4)
   (5) change-of-coordinate matrix $P_{B\leftarrow C}$ for bases B and C (4.7)

3. **Matrices.**
   (1) inverse matrices and their properties (2.2-2.3), Invertible Matrix Theorem characterizing invertible matrices by properties of columns, rows, rank, determinant
   (2) computing the inverse matrix via row reduction (2.2)
   (3) formula for inverse of 2×2-matrix (2.2)
   (4) Rank Theorem relating the rank and null space

4. **Linear maps.**
   (1) a linear map is determined by its images on a basis
   (2) matrix $T_{C\leftarrow B}$ of a linear map T with respect to bases B, C, obtaining the standard matrix $T_{E\leftarrow E}$ (for standard basis E) using change-of-coordinate matrices (1.9, 4.7)
   (3) matrix for rotation, reflection in $\mathbb{R}^2$ and $\mathbb{R}^3$ (cf. 1.9)
   (4) isomorphism between vector spaces, n-dimensional vector space is isomorphic to $\mathbb{R}^n$ (4.4)