# Math 2135 - Practice Midterm 2

**Except for problem 1, give full justifications and computations for all your answers!**

(1) True or false?
   (a) _____ The dimensions of $\operatorname{Col} A$ and $\operatorname{Row} A$ are the same for every matrix $A$.
   (b) _____ For $n \in \mathbb{N}$, a linear map $f\colon \mathbb{R}^n \to \mathbb{R}^n$ is bijective iff the kernel of $f$ is trivial.

(2) (a) List the conditions under which a function $f\colon V \to W$ for vector spaces $V, W$ over $\mathbb{R}$ is linear.
   (b) Let $A, B, C \in \mathbb{R}^{n\times n}$. Assume that $A$ is invertible. Show that if $AB = AC$, then $B = C$.

(3) Let $T\colon P_2 \to \mathbb{R}, p \mapsto p(3)$, be the map that evaluates a polynomial $p(t)$ at $t = 3$.
   (a) Show that $T$ is linear.
   (b) Determine the kernel of $T$, that is, $\ker T = \{p \in P_2 : T(p) = 0\}$, and the image of $T$, that is, $T(P_2)$.
   (c) Is $T$ injective, surjective, bijective?

(4) Let $B = (b_1, b_2)$ with $b_1 = \begin{bmatrix} -5 \\ 11 \\ 5 \end{bmatrix}$, $b_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ and $C = \left(\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right)$ be bases of a subspace $H$ of $\mathbb{R}^3$.
   (a) Compute the coordinates $[b_1]_C$ and $[b_2]_C$.
   (b) What is the change of coordinate matrix $P_{C\leftarrow B}$?
   (c) What is the change of coordinate matrix $P_{B\leftarrow C}$?

(5) Let $C = (1 + t, t + t^2, 1 + t^2)$ be a basis for $P_2$. Compute the coordinates $[p]_C$ for $p = 2 + t^2$.