Math 2135 - Assignment 8

Due October 23, 2026

(1) Give a basis for $\text{Nul}(A)$ and a basis for $\text{Col}(A)$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

(2) Give 2 different bases for

$$H = \text{Span}\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \}$$

(3) Let

$$A = \begin{bmatrix} -5 & 8 & 0 & -17 & -2 \\ 3 & -5 & 1 & 5 & 1 \\ 11 & -19 & 7 & 1 & 3 \\ 7 & -13 & 5 & -3 & 1 \end{bmatrix}.$$

Find bases and dimensions for $\text{Nul}\, A$, $\text{Col}\, A$, and $\text{Row}\, A$, respectively.

(4) True or false? Explain.
   (a) If B is an echelon form of a matrix A, then the pivot columns of B form a basis for the column space of A.
   (b) If B is an echelon form of a matrix A, then the nonzero rows of B form a basis for the row space of A.

(5) Let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 1 \\ 2.5 \\ -5 \end{bmatrix}$.
   (a) Find vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ such that $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{u}_1, \ldots, \mathbf{u}_k)$ is a basis for $\mathbb{R}^3$.
   (b) Find vectors $\mathbf{v}_1, \ldots, \mathbf{v}_\ell$ such that $(\mathbf{b}_3, \mathbf{v}_1, \ldots, \mathbf{v}_\ell)$ is a basis for $\mathbb{R}^3$.

   Prove that your choices for (a) and (b) form a basis.

(6) A 25×35 matrix A has 20 pivots. Find $\dim \text{Nul}\, A$, $\dim \text{Col}\, A$, $\dim \text{Row}\, A$, and $\text{rank}\, A$.

(7) True or false? Explain.
   (a) A basis of B is a set of linear independent vectors in V that is as large as possible.
   (b) If $\dim V = n$, then any n vectors that span V are linearly independent.
   (c) Every 2-dimensional subspace of $\mathbb{R}^2$ is a plane.

(8) Let $B = (1, t, t^2)$ and $C = (1, 1+t, 1+t+t^2)$ be bases of $\mathbb{P}_2$.
   (a) Determine the polynomials p, q with $[p]_B = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$ and $[q]_C = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$.
   (b) Compute $[r]_B$ and $[r]_C$ for $r = 3 + 2t + t^2$.

(9) Let $B = (b_1, \ldots, b_n)$ be a basis for a vector space V and consider the coordinate mapping $V \to \mathbb{R}^n,\ x \mapsto [x]_B$.
   (a) Show that $[c \cdot x]_B = c[x]_B$ for all $x \in V, c \in \mathbb{R}$.
   (b) Show that the coordinate mapping is onto $\mathbb{R}^n$.