

Math 2135 - Assignment 7

Due October 16, 2026

- (1) Explain why the following are not subspaces of $\mathbb{R}^2$. Give explicit counter examples for subspace properties that are not satisfied.
- (a) $U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, x \geq 0 \right\}$
 - (b) $V = \mathbb{Z}^2$ ($\mathbb{Z}$ denotes the set of all integers)
 - (c) $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, |x| = |y| \right\}$ ($|x|$ denotes the absolute value of x).
- (2) Let $A \in \mathbb{R}^{m \times n}$. Prove that $\text{Nul}(A)$ is a subspace of $\mathbb{R}^n$.
- (3) Which of the following are subspaces of the vector space $\mathbb{R}^{\mathbb{R}} = \{f: \mathbb{R} \rightarrow \mathbb{R}\}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$? Check all subspace properties or give one that is not satisfied.
- (a) $\{f: \mathbb{R} \rightarrow \mathbb{R} \mid f(0) = 1\}$
 - (b) $\{f: \mathbb{R} \rightarrow \mathbb{R} \mid f(3) = 0\}$
 - (c) $\{f: \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is continuous}\}$
- (4) Explain whether the following are true or false (give counter examples if possible):
- (a) Every vector space is a subspace of itself.
 - (b) Each plane in $\mathbb{R}^3$ is a subspace.
 - (c) Let U be a subspace of a vector space V . Any linear combination of vectors of U is also in V .
 - (d) Let $v_1, \dots, v_n$ be in a vector space V . Then $\text{Span}(v_1, \dots, v_n)$ is the smallest subspace of V containing $v_1, \dots, v_n$.
- (5) Are the vectors $\mathbf{v}_0 = 1$, $\mathbf{v}_1 = t$, $\mathbf{v}_2 = t^2$ in the vector space $\mathbb{R}^{\mathbb{R}} := \{f: \mathbb{R} \rightarrow \mathbb{R}\}$ linearly independent?
- (6) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = \left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right), B = \left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} \right), C = \left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

- (7) Give a basis for $\text{Nul}(A)$ and a basis for $\text{Col}(A)$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

- (8) Give 2 different bases for

$$H = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \right\}$$