

Math 2135 - Assignment 5

Due October 2, 2026

- (1) If defined, compute the following for the matrices

$$A = \begin{bmatrix} 2 & 1 & -4 \\ 3 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ -3 & 4 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$$

Else explain why the computation is not defined.

- (a) AB (b) BA (c) AC (d) $A + C$ (e) $AB + 2C$
- (2) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ first rotate points around the origin by 60° counter clockwise and then reflect points at the line with equation $y = x$. Give the standard matrix for T following these steps:
- (a) Recall the standard matrix A for the rotation R by 60° from class.
- (b) Determine the standard matrix B for the reflection S at the line with equation $y = x$ (a sketch will help).
- (c) Since T is the composition of S and R , compute the standard matrix C of T as the product of B and A . Careful about the order!
- (3) Continuation of (2): What is the standard matrix for $U: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which first reflects points at the line with equation $y = x$ and then rotates points around the origin by 60° counter clockwise? Compare T and U .
- (4) Prove for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$ that

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Hint: Multiply A with the given matrix and check the result.

- (5) A **diagonal matrix** A has all entries 0 except on the diagonal, that is,

$$A = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}.$$

Under which conditions is A invertible and what is A^{-1} ?

- (6) Let $A, B \in \mathbb{R}^{n \times n}$ be invertible. Show $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.
Hint: Multiply AB with the given matrix and check the result.
- (7) A matrix $C \in \mathbb{R}^{n \times m}$ is called a **left inverse** of a matrix $A \in \mathbb{R}^{m \times n}$ if $CA = I_n$ (the $n \times n$ identity matrix).
- (a) Show that if A has a left inverse C , then $Ax = b$ has at most one solution for any $b \in \mathbb{R}^m$.
- (b) Give an example of a matrix A that has a left inverse but is not invertible and a vector b such that $Ax = b$ has no solution.