Math 2135 - Practice Midterm 1

Except for problem 1, give full justifications and computations for all your answers!

(1) True or false?
   (a) If $Ax = b$ is inconsistent for some vector b, then A cannot have a pivot in every column.
   (b) If vectors $\mathbf{v}_1, \mathbf{v}_2$ are linearly independent and $\mathbf{v}_3$ is not in the span of $\mathbf{v}_1, \mathbf{v}_2$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is linear independent.
   (c) The range of $T\colon \mathbb{R}^n \to \mathbb{R}^m, x \mapsto Ax$, is the span of the columns of A.
   (d) If the first two columns of a matrix B are equal, then so are the first two columns of AB.
   (e) There exist square matrices A, B such that neither A nor B is 0 (the matrix with all entries 0) but $AB = 0$.
   (f) $f\colon \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+1 \\ 2y \end{bmatrix}$ is linear
   (g) The vectors $[1]$ and $[0]$ span $\mathbb{R}^1$.
   (h) If it is possible to reduce an augmented matrix $[A\ \mathbf{b}]$ to reduced echelon form, then $A\mathbf{x} = \mathbf{b}$ has at least one solution.

(2) Let
$$A = \begin{bmatrix} 0 & 3 & 1 & 2 \\ 1 & 4 & 0 & 7 \\ 2 & -1 & -3 & 8 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 5 \\ -8 \end{bmatrix}$$
   (a) Give the solution for $Ax = b$ in parametrized vector form.
   (b) Give vectors that span the null space of A.

(3) Let $T\colon \mathbb{R}^n \to \mathbb{R}^n, x \mapsto Ax$, be an injective linear map. Show that T is surjective as well.

(4) Is $\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$?

(5) Give the standard matrix for the reflection T of $\mathbb{R}^2$ on the line $2x + 3y = 0$.