Math 2135 - Assignment 11

Due November 14, 2025

You can use some computer algebra system to check your solutions for this assignment but have to show your calculations of characteristic polynomials, eigenvalues, etc, by hand.

(1) Let $A \in \mathbb{R}^{n\times n}$. Are the following true or false? Explain why:
  (a) If two rows or columns of A are identical, then $\det A = 0$.
  (b) For $c \in \mathbb{R}$, $\det(cA) = c \det A$.
  (c) If A is invertible, then $\det A^{-1} = \frac{1}{\det A}$.
  (d) A is invertible iff 0 is not an eigenvalue of A.

(2) Eigenvalues, -vectors and -spaces can be be defined for linear maps just as for matrices.

Let $h\colon V \to W$ be a linear map for vector spaces V, W over F. Show that the eigenspace for $\lambda \in F$,

$$E_{h,\lambda} := \{x \in V \ : \ h(x) = \lambda x\},$$

is a subspace of V.

(3) Are the following eigenvalues for the respective matrices? If so, give a basis for the corresponding eigenspace.

$$A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix}, \lambda = -5 \qquad B = \begin{bmatrix} 3 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}, \mu = 2$$

(4) Give all eigenvalues and bases for eigenspaces of the following matrices. Do you need the characteristic polynomials?

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

(5) Give the characteristic polynomial, all eigenvalues and bases for eigenspaces for $C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

(6) Compute eigenvalues and eigenvectors for $D = \begin{bmatrix} -1 & 4 & 1 \\ 6 & 9 & 2 \\ 0 & 0 & -3 \end{bmatrix}$.

(7) Let $A \in \mathbb{R}^{n\times n}$ with n eigenvalues $\lambda_1, \ldots, \lambda_n$ (repeated according to their multiplicities). Show that

$$\det A = \lambda_1 \cdot \lambda_2 \cdots \lambda_n$$

Hint: Note that the characteristic polynomial of A can be factored as

$$\det(A - \lambda I) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda).$$

Why? Check that the signs are correct.