Math 2135 - Assignment 7

Due October 17, 2025

(1) Which of the following are subspaces of the vector space $\mathbb{R}^{\mathbb{R}} = \{f : \mathbb{R} \to \mathbb{R}\}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$? Check all subspace properties or give one that is not satisfied.
   (a) $\{f : \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$
   (b) $\{f : \mathbb{R} \to \mathbb{R} \mid f(3) = 0\}$
   (c) $\{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$

(2) Explain whether the following are true or false (give counter examples if possible):
   (a) Every vector space is a subspace of itself.
   (b) Each plane in $\mathbb{R}^3$ is a subspace.
   (c) Let U be a subspace of a vector space V. Any linear combination of vectors of U is also in V.
   (d) Let $v_1, \ldots, v_n$ be in a vector space V. Then $\text{Span}(v_1, \ldots, v_n)$ is the smallest subspace of V containing $v_1, \ldots, v_n$.

(3) Are the vectors $\mathbf{v}_0 = 1$, $\mathbf{v}_1 = t$, $\mathbf{v}_2 = t^2$ in the vector space $\mathbb{R}^{\mathbb{R}} := \{f : \mathbb{R} \to \mathbb{R}\}$ linearly independent?

(4) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}), B = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\-1\\4\end{bmatrix}), C = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix})$$

(5) Show that the vectors $\mathbf{v}_0 = 1$, $\mathbf{v}_1 = 1 + t$, $\mathbf{v}_2 = 1 + t + t^2$ form a basis for the vector space P_2 of polynomials of degree ≤ 2.

(6) Give a basis for $\text{Nul}(A)$ and a basis for $\text{Col}(A)$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

(7) Give 2 different bases for

$$H = \text{Span}\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\3\\4\end{bmatrix}\}$$