Math 2135 - Assignment 5

Due October 3, 2025

You can check your results using Mathematica but do the calculations by hand and show them to receive credit.

(1) Prove for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$ that

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Hint: Multiply A with the given matrix and check the result.

(2) Are the following invertible? Compute the inverse if possible.

$$A = \begin{bmatrix} 2 & 1 \\ 4 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

(3) A **diagonal matrix** A has all entries 0 except on the diagonal, that is,

$$A = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}.$$

Under which conditions is A invertible and what is A^{-1}?

(4) Compute the inverse if possible:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$$

(5) Let $A, B \in \mathbb{R}^{n \times n}$ be invertible. Show $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.
Hint: Multiply AB with the given matrix and check the result.

(6) A matrix $C \in \mathbb{R}^{n \times m}$ is called a **left inverse** of a matrix $A \in \mathbb{R}^{m \times n}$ if $CA = I_n$ (the $n \times n$ identity matrix).

(a) Show that if A has a left inverse C, then $Ax = b$ has at most one solution for any $b \in \mathbb{R}^m$.

(b) Give an example of a matrix A that has a left inverse but is not invertible and a vector b such that $Ax = b$ has no solution.