

Math 2135 - Assignment 1

Due September 5, 2025

Solve all systems of linear equations by row reduction (Gaussian elimination) and give the solutions in parametric vector form.

- (1) Do the following 4 planes intersect in a point? Which?

$$\begin{aligned}x + 5y + 3z &= 16 \\2x + 10y + 8z &= 34 \\4x + 20y + 15z &= 67 \\x + 6y + 5z &= 21\end{aligned}$$

- (2) Add an equation of a line to the equation

$$2x + 3y = 4$$

such that the resulting system has (a) no solution, (b) exactly one solution, (c) infinitely many solutions.

- (3) Solve the system of linear equations with augmented matrix

$$\left[\begin{array}{cccc} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{array} \right]$$

- (4) Solve the system of linear equations with augmented matrix

$$\left[\begin{array}{ccccc} 2 & 2 & 1 & 8 & 2 \\ 2 & 0 & 0 & 8 & 2 \\ 2 & 6 & 3 & 8 & 1 \end{array} \right]$$

- (5) Let $A \in \mathbb{R}^{3 \times 2}$. Explain why $Ax = b$ cannot have a solution x for all $b \in \mathbb{R}^3$.

- (6) [1, cf. Section 1.5, Ex 17] Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

- (7) [1, cf. Section 1.5, Ex 11] Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form.

- (8) Are the following true or false? Explain your answers.

- (a) Any system of linear equations with strictly less equations than variables has infinitely many solutions.
- (b) Different sequences of elementary row reductions can transform one matrix to distinct matrices in row echelon form.
- (c) A consistent system has exactly one solution.
- (d) There exist inconsistent homogenous systems.
- (e) If a homogenous system has strictly less equations than variables, then it has infinitely many solutions.

REFERENCES

- [1] David C. Lay, Steven R. Lay, and Judi J. McDonald. Linear Algebra and Its Applications. Addison-Wesley, 6th edition, 2020.