Math 2135 Fall 2024 - Review for Finals

Numbers in parentheses refer to sections in Lay et al, Linear algebra and its applications, ed. 5.

Numbers in square brackets to assignments covering that topic.

1. **Systems of linear equations.**
   (1) coefficient and augmented matrix (1.4)
   (2) solving a linear system by row reduction, pivot columns, free variables, solution in parametrized vector form (1.2) [Ex 1.8]
   (3) solutions of homogenous vs. inhomogenous systems (1.5) [Ex 1.6, 2.8]
   (4) least squares solutions (6.5) [Ex 13.6, 13.7]

2. **Vector spaces.**
   (1) vector spaces and examples: tuples, functions, polynomials P_n (4.1)
   (2) subspaces: definition and examples (span, null space) (4.1) [Ex 7.1, 11.4]

3. **Basis of vector spaces.**
   (1) dimension, Basis Theorem (4.5)
   (2) reduce a spanning set to a basis (2.8, 4.3), extend a linear independent set to a basis (4.5) [Ex 8.2, 9.1]
   (3) bases and dimension for column space, row space, null space of a matrix (2.8, 4.3) [Ex 8.7, 9.2]
   (4) coordinates with respect to a basis B (2.9, 4.4) [Ex 10.3]
   (5) change of coordinate matrix $P_{B \leftarrow C}$ for bases B and C (4.7) [Ex 9.5]
   (6) orthogonal basis, coordinates via dot product (6.2), orthogonal projection (6.3) [Ex 13.1, 13.2]

4. **Matrices.**
   (1) matrix product and composition of linear maps (2.1) [Ex 4.7, 4.8]
   (2) inverse matrices and their properties, Invertible Matrix Theorem (2.2, 2.3) [Ex 6.8]
   (3) rank of a matrix (4.6) [Ex 10.5]
   (4) inverse matrix via row reduction (2.2) [Ex 6.1]
   (5) formula for inverse of 2×2-matrix (2.2) [Ex 5.6]
   (6) determinant via cofactor expansion (3.1) and via row reduction (3.2) [Ex 10.6, 11.1]
   (7) eigenvalues and eigenvectors of matrices (5.1), characteristic polynomials (5.2) [Ex 11.6, 11.7]
   (8) diagonalizing matrices, powers of matrices (5.3) [Ex 12.1, 12.2]

5. **Linear maps.**
   (1) a linear map is determined by its images on a basis (1.8) [Ex 4.1, 4.4]
   (2) matrix $T_{B\leftarrow C}$ of a linear map f with respect to bases B, C, standard matrix $T_{E\leftarrow E}$ (for standard basis E of $\mathbb{R}^n$) (1.9, 4.7) [Ex 9.4]
   (3) matrix for rotation, reflection in $\mathbb{R}^2$ and $\mathbb{R}^3$ (1.9) [Ex 4.7, 4.8, 9.6]
   (4) injective, surjective, bijective linear maps and connections with kernel, range (4.2) [Ex 4.5, 5.4, 6.6, 9.8]
   (5) isomorphism between vector spaces, n-dimensional vector space is isomorphic to $\mathbb{R}^n$ (4.4)