Math 2135 - Assignment 12

Due Nov 22, 2024

(1) Are the matrices A, B, C, D in (5), (6), (7) of assignment 11 diagonalizable? How?

(2) Let A be an $n \times n$-matrix. Are the following true or false? Explain why:
   (a) If A has n eigenvectors, then A is diagonalizable.
   (b) If a 4×4-matrix A has two eigenvalues with eigenspaces of dimension 3 and 1, respectively, then A is diagonalizable.
   (c) A is diagonalizable iff A has n eigenvalues (counting multiplicities).
   (d) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable.
   (e) Every triangular matrix is diagonalizable.

(3) Let A be the standard matrix for the reflection t of $\mathbb{R}^2$ on some line g throught the origin. What are the eigenvalues, eigenvectors and eigenspaces of A? Can A be diagonalized?
Hint: Consider what a reflection does to specific vectors.

(4) As the previous problem for a rotation r of $\mathbb{R}^2$ by an angle φ around the origin.
Hint: Consider $\varphi = 0, \pi$ separately.

(5) Consider a population of owls feeding on a population of squirrels. In month k, let o_k denote the number of owls and s_k the number of squirrels. Assume that the populations change every month as follows:

$$\begin{aligned} o_{k+1} &= 0.3o_k + 0.4s_k \\ s_{k+1} &= -0.4o_k + 1.3s_k \end{aligned}$$

That is, if there would be no squirrels to hunt, only 30% of the owls would survive to the next month; if there were no owls that hunted squirrels, then the squirrel population would grow by factor 1.3 every month.

Let $x_k = \begin{bmatrix} o_k \\ s_k \end{bmatrix}$. Express the population change from x_k to x_{k+1} using a matrix A. Diagonalize A.

(6) Continue the previous problem: Let the starting population be $x_1 = \begin{bmatrix} o_1 \\ s_1 \end{bmatrix} = \begin{bmatrix} 20 \\ 100 \end{bmatrix}$.
   (a) Give an explicit formula for the populations in month $k + 1$.
   (b) Are the populations growing or decreasing over time? By which factor?
   (c) What is ratio of owls to squirrels after 12 months? After 24 months? Can you explain why?