Math 2135 - Assignment 8

Due October 25, 2024

(1) Show that the vectors $\mathbf{v}_0 = 1$, $\mathbf{v}_1 = 1 + t$, $\mathbf{v}_2 = 1 + t + t^2$ form a basis for the vector space P_2 of polynomials of degree ≤ 2.

(2) Give a basis for $\mathrm{Nul}(A)$ and a basis for $\mathrm{Col}(A)$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$

(3) Give 2 different bases for

$$H = \mathrm{Span}\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \}$$

(4) Let $B = (b_1, \dots, b_n)$ be a basis for a vector space V and consider the coordinate mapping $V \to \mathbb{R}^n,\ x \mapsto [x]_B$.

(a) Show that $[c \cdot x]_B = c[x]_B$ for all $x \in V, c \in \mathbb{R}$.

(b) Show that the coordinate mapping is onto $\mathbb{R}^n$.

(5) Let $B = (\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix})$ be a basis of $\mathbb{R}^2$.

(a) Find vectors $u, v \in \mathbb{R}^2$ with $[u]_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [v]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(b) Compute the coordinates relative to B of $w = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(6) Let $B = (1, t, t^2)$ and $C = (1, 1 + t, 1 + t + t^2)$ be bases of $\mathbb{P}_2$.

(a) Determine the polynomials p, q with $[p]_B = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$ and $[q]_C = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$.

(b) Compute $[r]_B$ and $[r]_C$ for $r = 3 + 2t + t^2$.

(7) Let

$$A = \begin{bmatrix} -5 & 8 & 0 & -17 & -2 \\ 3 & -5 & 1 & 5 & 1 \\ 11 & -19 & 7 & 1 & 3 \\ 7 & -13 & 5 & -3 & 1 \end{bmatrix}.$$

Find bases and dimensions for $\mathrm{Nul}\,A$, $\mathrm{Col}\,A$, and $\mathrm{Row}\,A$, respectively.

(8) True or false? Explain.

(a) If B is an echelon form of a matrix A, then the pivot columns of B form a basis for the column space of A.

(b) If B is an echelon form of a matrix A, then the nonzero rows of B form a basis for the row space of A.

(c) If $\dim V = n$, then any n vectors that span V are linearly independent.

(d) Every 2-dimensional subspace of $\mathbb{R}^3$ is a plane.