Math 2135 - Assignment 7

Due October 18, 2024

(1) Explain why the following are not subspaces of $\mathbb{R}^2$. Give explicit counter examples for subspace properties that are not satisfied.

(a) $U = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, x \geq 0\}$

(b) $V = \mathbb{Z}^2$ ($\mathbb{Z}$ denotes the set of all integers)

(c) $W = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, |x| = |y|\}$

(2) Which of the following are subspaces of the vector space $\mathbb{R}^{\mathbb{R}} = \{f \colon \mathbb{R} \to \mathbb{R}\}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$? Check all subspace properties or give one that is not satisfied.

(a) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$

(b) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(3) = 0\}$

(c) $\{f \colon \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$

(3) Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors in a vector space V. Show that $U := \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a subspace of V.

(4) Let $A \in \mathbb{R}^{m \times n}$. Prove that $\text{Nul}(A)$ is a subspace of $\mathbb{R}^n$.

(5) Explain whether the following are true or false (give counter examples if possible):

(a) Every vector space is a subspace of itself.

(b) Each plane in $\mathbb{R}^3$ is a subspace.

(c) Let U be a subspace of a vector space V. Any linear combination of vectors of U is also in V.

(d) Let $v_1, \ldots, v_n$ be in a vector space V. Then $\text{Span}(v_1, \ldots, v_n)$ is the smallest subspace of V containing $v_1, \ldots, v_n$.

(6) Are the vectors $\mathbf{v}_0 = 1$, $\mathbf{v}_1 = t$, $\mathbf{v}_2 = t^2$ in the vector space $\mathbb{R}^{\mathbb{R}} := \{f \colon \mathbb{R} \to \mathbb{R}\}$ linearly independent?

(7) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}), B = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}), C = (\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix})$$

(8) Give a basis for $\text{Nul}(A)$ and a basis for $\text{Col}(A)$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}$$