Math 2135 - Assignment 1

Due September 6, 2024

Solve all systems of linear equations by row reduction (Gaussian elimination) and give the solutions in parametric vector form.

(1) Do the following 4 planes intersect in a point? Which?

$$\begin{aligned} x + 5y + 3z &= 16 \\ 2x + 10y + 8z &= 34 \\ 4x + 20y + 15z &= 67 \\ x + 6y + 5z &= 21 \end{aligned}$$

(2) Add an equation of a line to the equation

$$2x + 3y = 4$$

such that the resulting system has (a) no solution, (b) exactly one solution, (c) infinitely many solutions.

(3) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(4) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 2 & 2 & 1 & 8 & 2 \\ 2 & 0 & 0 & 8 & 2 \\ 2 & 6 & 3 & 8 & 1 \end{bmatrix}$$

(5) Let $A \in \mathbb{R}^{3\times 2}$. Explain why $Ax = b$ cannot have a solution x for all $b \in \mathbb{R}^3$.

(6) [1, cf. Section 1.5, Ex 17] Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

(7) [1, cf. Section 1.5, Ex 11] Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $Ax = b$ and $Ax = \mathbf{0}$. Express both solution sets in parametric vector form.

(8) Are the following true or false? Explain your answers.

(a) Any system of linear equations with strictly less equations than variables has infinitely many solutions.
(b) Different sequences of elementary row reductions can transform one matrix to distinct matrices in row echelon form.
(c) A consistent system has exactly one solution.
(d) There exist inconsistent homogenous systems.
(e) If a homogenous system has strictly less equations than variables, then it has infinitely many solutions.

References

[1] David C. Lay, Steven R. Lay, and Judi J. McDonald. Linear Algebra and Its Applications. Addison-Wesley, 5th edition, 2015.