Math 2001 - Review for Midterm 2

Counting.

(1) Lists: with/without repetitions, permutations - factorials (3.1,, 3.2), subsets - binomials (3.3), integer solutions of $x_1 + x_2 + \cdots + x_n = k$
(2) Binomial Theorem: Pascal's triangle (3.4)
(3) Inclusion-Exclusion (3.5)

Modular arithmetic.

(1) Integers: divisibility, division algorithm, gcd, lcm, extended Euclidean algorithm, Bezout's identity and coefficients
(2) congruences (5.2), integers mod n (11.4)

Proof methods.

(1) direct proof (4.2, 4.3), contrapositive proof (5), proof by contradiction (6), proof of if-and-only-if statements

Some practice problems.

(1) (a) Compute $3 - 8 \mod 11$ and $16 \cdot 20 \mod 11$.
 (b) Compute $\gcd(111, 33)$ and its Bezout coefficients.
(2) Prove that $\sqrt[3]{2}$ is irrational.
(3) Let $a, b \in \mathbb{Z}$. Show that $a \equiv b \mod 6$ if and only if $a \equiv b \mod 2$ and $a \equiv b \mod 3$.
(4) Let $a \in \mathbb{Z}$. Show that $\gcd(a, a+1) = 1$.
(5) Give the first sentence (the assumptions) for the proofs of the following statements:
 (a) Let $a, b \in \mathbb{Z}$. If $a|b$ and $b|a$, then $a = b$ or $a = -b$. (direct proof)
 (b) Let $a, b \in \mathbb{Z}$. If $(a+b)^2 = a^2 + b^2$, then $a = 0$ or $b = 0$. (contrapositive proof)
 (c) Let $x, y \in \mathbb{Z}$. If $4|x^2 + y^2$, then x and y are even. (proof by contradiction)