Math 2001 - Review problems for Midterm 1

(1) Use the set-builder notation to describe:
   (a) $A = \{4, 16, 36, 64, 100\}$
   (b) $B =$ the set of subsets of $\mathbb{N}$ of odd order

(2) (a) Check whether for all sets A, B

$$(A \cup B) - B = A$$

You may use Venn diagrams.

   (b) Prove without Venn diagrams that for all sets A, B in a universe U:

$$\bar{A} - \bar{B} = B - A.$$

(3) Simplify:
   (a) $\bigcup_{i=0}^{4} [i, 2i + 1]$
   (b) $\bigcap_{n \in \mathbb{N}} \{x \in \mathbb{Z} : x \text{ is a multiple of } n\}$

(4) Is it true that for all statements P, Q

$$(P \Rightarrow Q) \wedge P = Q$$

Prove it or give a counter-example.

(5) Write using quantifiers and logical operations:
   (a) The square of any real number is non-negative.
   (b) There exists an integer x such that $x^y = x$ for all integers y.

(6) Negate without using the phrase "It is not true that..." and without "$\sim$":
   (a) $\forall n \in \mathbb{N}\ \exists A \in P(\mathbb{N}) : |A| > n$
   (b) $\forall$ polynomial $p\ \exists x \in \mathbb{R} :\ p(x) = 0$ or p is constant
   (c) $x + y = 0$ and $x - y = 0$ iff $x = 0$ and $y = 0$.