Math 2001 - Assignment 10

Due April 6, 2018

The first 2 problems are meant to be revision for the midterm. Do them before Wednesday.

(1) (a) How many permutations of the alphabet $a, \ldots, z$ contain the word "fish"?
   (b) How many permutations of the alphabet do not contain any of the words "fish", "rat" or "bird"?

(2) A regular poker card set has 4 suits and 13 cards for each suit.
   (a) How many sets of 5 cards (out of 52) are there with 4 cards of one kind?
   (b) How many sets of 5 cards (out of 52) are there with all cards of the same suit?

(3) Prove by induction that for every $q \in \mathbb{R}$ with $q \neq 1$ and for every $n \in \mathbb{N}_0$:

$$1 + q^1 + q^2 + \cdots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

(4) [1, Chapter 10, exercise 8] Show that for every $n \in \mathbb{N}$:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

(5) Let $n \in \mathbb{N}$ and let $A_1, \ldots, A_n$ be sets in some universe U. Show by induction that

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \bar{A}_1 \cap \bar{A}_2 \cap \ldots \bar{A}_n$$

Hint: for the base case consider $n = 1$ and $n = 2$.

(6) Show by induction that for every natural number $n \geq 4$:

$$2^n \geq n^2$$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`