

# Math 2001 - Assignment 9

Due March 23, 2018

Please put the solutions to problems (1), (2) on a separate piece of paper from the rest.

- (1) [1, Chapter 6, exercise 8] Prove by contradiction: Let  $a, b, c \in \mathbb{Z}$ . If  $a^2 + b^2 = c^2$ , then  $a$  or  $b$  is even.
- (2) Prove for all  $x, y \in \mathbb{R}$ :  
If  $x$  is rational and  $xy$  is irrational, then  $y$  is irrational.
- (3) Compute:
  - (a)  $3 \cdot 4 \pmod{7}$
  - (b)  $2 - 9 \pmod{11}$
  - (c)  $2^6 \pmod{9}$
  - (d) Solve for  $x \in \mathbb{Z}$ :  $13x \equiv 3 \pmod{31}$   
Hint: First solve the equation  $13x + 31y = 3$  using the extended Euclidean algorithm.
- (4) Prove: Let  $a, b, c, d \in \mathbb{Z}$  and  $n \in \mathbb{N}$ . If  $a \equiv b \pmod{n}$  and  $c \equiv d \pmod{n}$ , then  $a + c \equiv b + d \pmod{n}$ .
- (5) [1, Chapter 10, exercise 2] Show by induction that for every  $n \in \mathbb{N}$ :

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

## REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013.  
Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>