Math 2001 - Assignment 8

Due March 16, 2018

(1) Read Section 5.3 in [1].

(2) Solve the following for $u, v \in \mathbb{Z}$:
(a) $33u + 10v = -5$ (b) $44u + 10v = 5$

(3) Complete the proof from class that $\gcd(a, b) = \gcd(a - qb, b)$ for all $a, b, q \in Z$ with not both a and b equal 0.
Assume $d|a - qb$ and $d|b$. Show that $d|a$ and $d|b$.

(4) Let $a, b, c \in \mathbb{Z}$ with a, b not both 0. Show that

$$\exists u, v \in \mathbb{Z}\colon\ u \cdot a + v \cdot b = c \text{ iff } \gcd(a, b)|c.$$

Hint: There are 2 implications to show:
(a) If $u \cdot a + v \cdot b = c$, then $\gcd(a, b)|c$.
(b) If $\gcd(a, b)|c$, then there are $u, v \in \mathbb{Z}$ such that $u \cdot a + v \cdot b = c$.
Use Bezout's identity!

(5) Two integers have the *same parity* if both are even or both are odd. Otherwise they have *opposite parity*.
Let $a, b \in \mathbb{Z}$. Show that if $a + b$ is even, then a, b have the same parity.
Hint: Use a contrapositive proof.

(6) Complete the following proof of **Euclid's Lemma:**
Let p be a prime, $a, b \in \mathbb{Z}$. If $p|ab$, then $p|a$ or $p|b$.

Proof: Assume ______________ but $p \nmid a$. We will show $p|b$.
By Bezout's identity we have $u, v \in \mathbb{Z}$ such that

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \gcd(a, p).$$

Since p is ______________ and $p \nmid a$, we have $\gcd(a, p) =$_____.
Hence

$$ua + vp = \_\_\_\_\_.$$

Multiplying this equation by _____ yields

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = b$$

Since $p|$______________ and $p|$______________, we have a multiple of p on the left hand side of this equation. Thus __________.
$\square$

Please hand in this sheet of paper with your solution of 6.

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013.