Math 2001 - Assignment 7

Due March 9, 2018

Please hand in problems 4,5,6 on a separate piece of paper. Be careful to write down every step in their proofs and reduce every statement to definitions or other statements that were already proved in class.

(1) In how many different ways can n students (called $A, B, ...$) line up in a queue for the cafeteria such that
  (a) students A and B stand next to each other?
  (b) students A and B do not stand next to each other?

(2) How many different seating arrangements are there on a round table with n seats?

Since a round table has no beginning or end, two arrangements are the same if one is obtained from the other by rotation, e.g., the following are considered equal:

(3) Compute $\gcd(a, b)$ and the Bezout coefficients using the Euclidean algorithm for the following numbers. Then find $\text{lcm}(a, b)$.
  (a) $a = 85, b = 25$
  (b) $a = 57, b = 24$

(4) Show that if x is an odd integer, then 8 divides $x^2 - 1$.
Recall: x is odd if it is of the form $x = 2y + 1$ for some $y \in \mathbb{Z}$.

(5) Let x be an integer. Show that if x^2 is odd, then x is odd.
Hint: Use contraposition.

(6) Two integers have the *same parity* if both are even or both are odd. Otherwise they have *opposite parity*.

Let $a, b \in \mathbb{Z}$. Show that if a, b have the same parity, then $a + b$ is even.
Hint: Use cases.