Math 2001 - Assignment 4

Due February 16, 2018

(1) Are the following equalities true for all statements P, Q?
   (a) $P \wedge \sim P =$ False
   (b) $\sim (P \Leftrightarrow Q) \;\; = \;\; \sim P \Leftrightarrow \sim Q$
(2) [1, Section 2.7]: Exercises 4,6,9,10
(3) Formulate the following sentences using quantifiers and logical operations. Are they true?
   (a) For all integers n we have that $n(n+1)$ is even.
   (b) If x^2 is rational, then so is x.
   (c) There exists a real number z such that $x + z = x$ for every real x.
   (d) Every real number is smaller than some integer.
(4) Negate the following sentences:
   (a) $xy = 0$ iff $x = 0$ or $y = 0$
   (b) The derivative of a polynomial function f is 0 iff f is constant.
   (c) $\exists x \in \mathbb{R} : x^2 = -1$
   (d) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer
(5) Are the following sentences true? Negate them:
   (a) There exists a right triangle that is not isosceles.
   (b) $\forall x \in \mathbb{R}^+ \; \exists n \in \mathbb{N} : \; \frac{1}{n} \leq x$.
   (c) $\exists m \in \mathbb{N} \; \forall p \in \mathbb{N} : p$ prime $\Rightarrow p \leq m$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`