Math 2001 - Assignment 2

Due February 2, 2018

(1) Describe the following using set builder notation:
   (a) $A =$ the set of points in $\mathbb{R}^2$ on the line through $(2, 3)$ that is parallel to the y-axis
   (b) $B =$ the set of points $(x, y) \in \mathbb{R}^2$ on the line through $(1, 2)$ and $(3, 4)$
   (c) $C =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0, 0)$ and radius 2

(2) For $U := \{x \in \mathbb{Z} : 1 \leq x \leq 8\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{x \in U : x \text{ is even }\}$, and $C = \{x \in U : x \geq 4\}$ compute:
   (a) $A \cap C_U(B)$ (b) $A \cup (B \cap C)$ (c) $(A - B) \cup B$

(3) Simplify and justify your answers:
   (a) $\bigcup_{n\in\mathbb{N}}(0, n]$ (b) $\bigcap_{n=1}^{3}\{nz : z \in \mathbb{Z}\}$ (c) $\bigcup_{A\in P(\mathbb{N})} A$

(4) Are the following true for all sets A, B in a universe U?
   (a) $A - B = B - A$
   (b) $A \cup B \subseteq (A \cap \bar{B}) \cup (B \cap \bar{A})$

   Consider Venn diagrams first and then either write a proof that the equations hold or give an example where they fail.

(5) Show that for all sets A, B, C

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

   without Venn diagrams.

   Recall that we already showed that the lefthand side is contained in the the righthand side. So it only remains to write a proof for the converse,

$$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C).$$

(6) Show for all sets A, B in the universe U:

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \text{(de Morgan's law)}$$

   First use Venn diagrams. Then write down a proof.