COMBINATORICS

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1. Combinatorial functions:

$n! = n \cdot (n-1) \cdots 2 \cdot 1$

$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$

2. Counting the most common objects:

A k-list is a list of length k.
A k-subset is a set of size k.

objects	**number**
k-lists with entries from $\{1, \ldots, n\}$ repetitions allowed	n^k
k-lists with entries from $\{1, \ldots, n\}$ $k \leq n$ repetitions not allowed	$\frac{n!}{(n-k)!}$
permutations of $\{1, \ldots, n\}$	$n!$
k-subsets of $\{1, \ldots, n\}$	$\binom{n}{k}$

3. Number of integer solutions of

$$x_1 + x_2 + \cdots + x_n = k$$

with $x_i \geq 0$ for all i:
There are $\binom{n+k-1}{k}$ solutions for the previous equation.

4. Inclusion-Exclusion:

For finite sets $A_1, \ldots, A_n$,

$$\begin{aligned} |A_1 \cup \ldots \cup A_n| = & |A_1| + \cdots + |A_n| \\ & - |A_1 \cap A_2| - |A_1 \cap A_3| - \cdots - |A_{n-1} \cap A_n| \\ & + |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \cdots + |A_{n-2} \cap A_{n-1} \cap A_n| \\ & \vdots \\ & + (-1)^{n-1} |A_1 \cap A_2 \cap \ldots \cap A_n| \end{aligned}$$

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