

Math 2001 - Practice Midterm 1

- (1) Use the Axiom of Replacement or Specification to describe:
- (a) $C = \{1, 9, 25, 49, 81\}$
 - (b) $D =$ the set of subsets of \mathbb{Z} of even size
- (2) Prove without Venn diagrams that for all sets A, B in a universe U :

$$\overline{A - B} = B - A$$

- (3) Let $P(A)$ denote the power set of A . Is the following true for all sets A, B ?

$$P(A \times B) \subseteq P(A) \times P(B)$$

Prove it or give a counter-example.

- (4) Write using quantifiers and logical operations:
- (a) The square of any real number is non-negative.
 - (b) There exists an integer x such that $x^y = x$ for all integers y .
- (5) Which of the following are true? Explain why or why not.
- (a) $\forall x \in \mathbb{Z} \forall y \in \mathbb{Z} : xy = y$
 - (b) $\forall x \in \mathbb{Z} \exists y \in \mathbb{Z} : xy = y$
 - (c) $\exists y \in \mathbb{Z} \forall x \in \mathbb{Z} : xy = y$
 - (d) $\exists x \in \mathbb{Z} \exists y \in \mathbb{Z} : xy = y$
- (6) Negate without using the phrase “It is not true that...” and without “ \sim ”:
- (a) $\forall n \in \mathbb{N} \exists A \in P(\mathbb{N}) : |A| > n$
 - (b) \forall polynomial $p \exists x \in \mathbb{R} : p(x) = 0$ or p is constant
 - (c) $x + y = 0$ and $x - y = 0$ iff $x = 0$ and $y = 0$.