

# Math 2001 - Writing project 3

First draft due November 12, final draft November 19, 2025

The following **writing project** will be graded on clarity and correctness and should be typed in LaTeX.

**Problem.** You want to split a stack of  $n$  boxes ( $n \in \mathbb{N}$ ), one box on top of the other, into  $n$  stacks of height 1.

- In each move, you can split a single stack, say of height  $n$ , into 2 stacks, say of heights  $a$  and  $b$  ( $a, b \in \mathbb{N}$ ) with  $a + b = n$ . This move has score  $ab$ .
- Then you can split one of the new stacks into two of smaller heights, say  $u, v$ , and add  $uv$  to your previous score.
- Repeat splitting single stacks and adding the product of heights of the new stacks to the previous score until you have  $n$  stacks of height 1.

What is the maximum score possible starting with a stack of height  $n$ ?

Your write up should include the following:

- (1) A section describing the problem.
- (2) A theorem stating the main result.
- (3) A proof of the theorem.
- (4) Give precise arguments for all your statements.