Math 2001 - Assignment 13

Due December 5, 2025

(1) (a) Give domain, codomain, and range of $f\colon \mathbb{Z} \to \mathbb{N}$, $x \mapsto x^2+1$. What is $f(3)$?
    (b) Is f one-to-one, onto, bijective?
    (c) Determine $f(\{2x \;:\; x \in \mathbb{Z}\})$ and $f^{-1}(\{1,2,3,\ldots,10\})$.

(2) Give examples for
    (a) a function $f\colon \mathbb{N} \to \mathbb{N}$ that is not injective but surjective;
    (b) a function $g\colon \{1,2,3\} \to \{1,2\}$ that is neither injective nor surjective;
    (c) a bijective function $h\colon \{1,2,3\} \to \{1,2\}$.

(3) Let $f\colon A \to B, g\colon B \to C$. Show that
    (a) If $g \circ f$ is injective, then f is injective.
    (b) If $g \circ f$ is surjective, then g is surjective.

    Hint: Use contrapositive proofs.

    Give examples for f, g on $A = B = C = \mathbb{N}$ such that
    (c) $g \circ f$ is injective but g is not injective;
    (d) $g \circ f$ is surjective but f is not surjective.

(4) (a) Show that

$$f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}, x \mapsto \frac{2x+1}{x-1}$$

    is bijective.
    (b) Determine f^{-1}.

(5) Try to you find an inverse for $f\colon \mathbb{R} \to \mathbb{R}^+$, $x \mapsto e^{x^3+1}$. Is f bijective?

(6) Let U be a set, and let c be the function on the power set of U that maps every set to its complement in U, i.e.,

$$c\colon P(U) \to P(U), X \mapsto \bar{X}.$$

Determine c^{-1} if it exists.